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The Characteristic Impedance of Rectangular Transmission Lines with Thin Center Conductor and Air Dielectric

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***Abstract*—The characteristic impedance of large-scale rectangular strip transmission line facilities used for such purposes as EMI susceptibility testing, biological exposures, etc., is discussed. These lines are characterized by a thin center conductor and an air dielectric. Impedance data obtained by earlier workers, using different analytical and numerical techniques, are reviewed and compared. Exact data are available for the problem involving a center conductor of zero thickness, while for the center conductor of finite thickness, data are available which are accurate to less than 1.25 percent.**

I. Introduction

Rectangular coaxial transmission lines which contain a propagating transverse electromagnetic (TEM) field are finding increasing application in such areas as EM susceptibility and emissions testing, biological effects of RF exposure, and calibration of radiation survey meters and electric field probes. Such lines possess an air dielectric with a thin center conductor, thereby maximizing the test space available between conductors. Crawford [1] has discussed the properties of such lines as well as their advantages, and has described a family of TEM "cells" constructed at the National Bureau of Standards. A similar transmission line of this type, used for purposes of exposing monkeys as well as large phantoms to HF-band (10–30-MHz) radiation fields has also been described [2], [3]. Others [4] have used much smaller rectangular lines of this type to investigate the interaction of microwaves with isolated nerve cells at frequencies up to 3 GHz. The use of such lines for calibration of radiation survey (hazard) meters as well as electric and magnetic field probes in the VHF and UHF bands has been discussed by Crawford [5], Baird [6], and Aslan [7]. A series of these transmission lines is now manufactured commercially by Instruments for Industries, Inc., Farmingdale, NY, and has been termed "Crawford Cells" by the manufacturer.

The characteristic impedance of such transmission lines has been quoted by Crawford [1] in terms of the fixed dimensions of the line's cross section (see Fig. 1 for notation) as well as an unknown fringing capacitance per unit length C_f'.

$$Z_0 = \frac{376.73}{4[w/(b-t)+C_f'/\epsilon]}\cdots \qquad (1)$$

where $\epsilon = 8.8542\times10^{-12}$ F/m, assuming an air dielectric. Crawford used time-domain reflectometry methods to ex-

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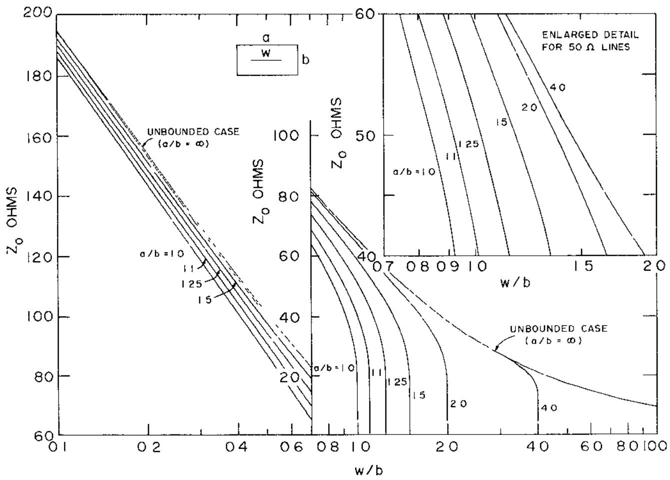

Fig. 1. Characteristic impedance for rectangular strip line with center conductor of zero thickness.

perimentally determine the unknown fringing capacitance. Such methods obviously do not make for a rapid and straightforward design procedure since they require, of necessity, a "cut-and-try" approach.

There exists, in fact, a considerable body of literature which deals with analytical and numerical solutions to the rectangular transmission line problem, thus obviating the need for experimental design. This paper attempts to review the previous work on this subject and compares data obtained using different analytical and numerical methods. Much of the existing data is in a form that is of little practical value to a design engineer. For this reason, convenient design curves have been compiled, which should assist researchers in the rapid design of large-scale transmission line facilities suitable for EMI susceptibility testing, biological exposures, or instrument calibration, etc.

II. Review of Previous Work

Rectangular lines may generally be classified according to the thickness of the center conductor, relative to the ground plane separation t/b. Rectangular lines with thin center conductors, $t/b \leqslant 0.2$, are sometimes termed "strip" lines while those having a thick center conductor (rectangular bar) with $0.2 < t/b < 1$ are usually called rectangular coaxial lines. This paper deals primarily with the "thin" class of rectangular lines where $t/b \leqslant 0.1$.

Almost all of the analytical solutions to the rectangular line problem are based on the closely related problem involving coupled coplanar strips between infinite ground planes [8]–[13]. Owing to the virtually identical patterns of flux distribution, the fringing or "corner" capacitance in the rectangular line is the same as that which exists in the coupled-strip problem during odd-mode excitation (C'_{fo}).

A. Zero Thickness Center Conductor

Using a conformal transformation technique, Cohn [8] obtained an exact solution to the coupled-strip problem with center conductors of zero thickness. By letting $w/b \rightarrow \infty$, (refer to Fig. 1 for dimensional notation) he then derived an expression for the odd-mode fringing capacitance:

$$\frac{C'_{fo}}{\epsilon} = \frac{2}{\pi} \ln\left[1 + \coth \pi\left(\frac{a-w}{2b}\right)\right] \cdots . \qquad (2)$$

The accuracy of (2) is satisfactory (less than 2 percent) provided that interaction effects between the two edges of the center conductor are not significant. Cohn [8] showed that such effects could be neglected, provided that the restriction $w/b \geqslant 0.35$ is maintained.

Recently, Tippet and Chang [9] have obtained a rigorous and exact solution to the rectangular line problem with zero-thickness center conductor that is not based on the related coupled-strip problem. The expression for Z_0 derived by Tippet and Chang is as follows:

$$Z_0 = 188.37 \frac{K(\lambda')}{K(\lambda)} \cdots \qquad (3)$$

where $K(\lambda)$ and $K(\lambda')$ are complete elliptic integrals of the first kind, and λ' is defined in terms of Jacobian elliptic functions as follows:

$$\lambda' = k'\left(\frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}\right)^2, \qquad \text{with } \lambda = \sqrt{1-\lambda'^2} \cdots \qquad (4)$$

where

$$\xi = K(k')\left(\frac{a-w}{2b}\right) \cdots \qquad (5)$$

and the modulus k' is derived from the identity

$$\frac{K(k)}{K(k')} = \frac{2a}{b} \cdots . \qquad (6)$$

The authors show how the exact expression for total capacitance per unit length reduces to an approximate form representing the inter-plate capacitance, w/b together with the fringing capacitance C'_{fo}, plus a correction factor ΔC which accounts for the interaction between the two edges of the center plate. An approximation for ΔC is given which is valid for $w/b \geqslant 0.1$, provided $k' \cong 1$ $(a/b \rightarrow \infty)$.

$$\frac{\Delta C}{\epsilon} \cong \frac{2}{\pi} \ln\left[\frac{8}{(1+\sqrt{\lambda})^2(1+\lambda)}\right] \cdots \qquad (7)$$

where

$$\lambda \cong \left[1 - \exp^{-2\pi w/b}\right]^{1/2}.$$

Equation (1) is modified as follows to include the correction factor:

$$Z_0 = \frac{376.73}{4\left[w/b + C'_{fo}/\epsilon\right] - \Delta C/\epsilon} \cdots . \qquad (8)$$

Fig. 1 shows characteristic impedance data for the case of a center conductor with zero thickness. These plots were obtained using (1) and (2) over the range $w/b \geqslant 0.5$. Over the range $0.1 \leqslant w/b < 0.5$, the correction factor ΔC becomes significant and must be included in the calculations of Z_0. For the unbounded case $a/b = \infty$, the correction factor can be computed directly using (7). For the remaining cases considered ($a/b = 1.0$, 1.1, etc.) reasonably accurate estimates of ΔC were derived using the data published in Fig. 2 of Riblet [10], where ΔC is four times the C_{fo} given by Riblet. In Fig. 1, it is readily apparent that the impedance plots become straight lines in the region $w/b<0.5$. Furthermore, it is apparent that Z_0 is almost independent of a/b as $w/b \to 0.1$; this means that the vertical side walls have little influence on the fringing fields when the center conductor is narrow. Some enlarged details for transmission lines having design impedances close to the standard 50-Ω value frequently used in practice are included as an insert to Fig. 1.

In Sections V and VI of a paper dealing specifically with the rectangular line problem, Chen [11] recognized the applicability of Cohn's earlier results [8] to this problem. Although he gives formula (2) as derived by Cohn, he fails to stress the important w/b restrictions on the applicability of (2), imposed by Cohn. In Section VI, Chen considers the special case of the rectangular line with narrow center conductor ($w/a \leqslant 0.25$). In this case, it is claimed that the problem reduces to that of the unbounded strip line ($a/b=\infty$) in which Z_0 is independent of a/b.

$$Z_0=\frac{376.73}{4\left[w/b+(2/\pi)\ln(2)\right]}\cdots. \tag{9}$$

This assumption, as noted earlier, is essentially correct. However, the impedance values derived from (9) do not agree with the data of Fig. 1, owing to Chen's neglect of the edge-interaction correction factor.

B. Center Conductor with Finite but Small Thickness

To date, there appears to be no general solution available for rectangular lines with center conductor of finite thickness. A solution can be found in terms of degenerate hyperelliptic integrals, but these functions have not been well tabulated. However, reasonably accurate solutions have also been obtained using estimates of the fringing capacitance derived from the coupled-strip problem with center conductors of finite thickness. Cohn [8] quotes the fringing capacitance for this case in terms of C'_{fo} given in (2) and additional fringing capacitance formulas applicable to shielded strip transmission lines with thin center conductors.

$$C'_{fo}\left(t/b,\frac{a-w}{b}\right)=\frac{C'_f(t/b)}{C'_f(o)}\cdot C'_{fo}\left(o,\frac{a-w}{b}\right)\cdots. \tag{10}$$

The reader is referred to an earlier paper of Cohn's [14] for data on $C'_f(t/b)$ and $C'_f(o)$. Chen [11] adopted the same approach and, based on some classical work of Sir J. J. Thomson, derived the following expression for the fringing capacitance:

$$\frac{C'_f}{\epsilon}=\frac{1}{\pi\ln(2)}\left\{\frac{b}{b-t}\ln\left(\frac{2b-t}{t}\right)+\ln\left[\frac{t(2b-t)}{(b-t)^2}\right]\right\}\cdot\ln\left[1+\coth\pi\left(\frac{a-w}{2b}\right)\right]\cdots. \tag{11}$$

No restrictions on the applicability of (11) are given by Chen. A somewhat different approximation was obtained by Joines [12]:

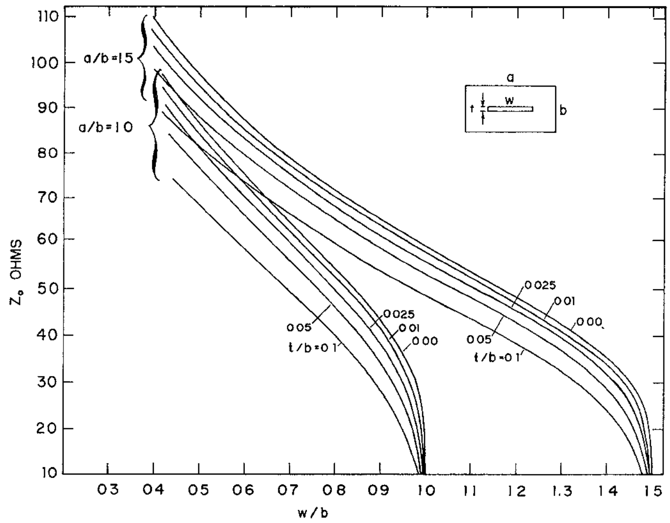

Fig. 2. Characteristic impedance for rectangular strip lines with center conductor of finite thickness ($t/b \leqslant 0.1$).

$$\frac{C'_f}{\epsilon}=\frac{1}{\pi}\left\{\frac{b}{b-t}\ln\left(\frac{2b-t}{t}\right)+\ln\left[\frac{t(2b-t)}{(b-t)^2}\right]+\ln\left[\coth\frac{\pi}{2}\left(\frac{a-w}{b-t}\right)\right]\right\}\cdots,\quad \text{valid for } \frac{a-w}{2b}\geqslant 0.4 \tag{12a}$$

and

$$\frac{C'_f}{\epsilon}=\frac{2b}{\pi(b-t)}\ln\left[1+\coth\frac{\pi}{2}\left(\frac{a-w}{b-t}\right)\right]+\frac{t}{a-w}\cdots,\quad \text{valid for } \frac{a-w}{2b}<0.4. \tag{12b}$$

Joines quotes the accuracy of these expressions as being within 2 percent as long as Cohn's restriction $w/(b-t)\geqslant 0.35$ is satisfied. Joines did not attempt to compute a correction factor for the edge interaction so that these results are only valid for the restricted range of w/b (See Fig. 2 for impedance data derived using (12)).

Getsinger [13] has obtained an exact solution to the problem involving coupled strips of infinite width ($w/b=\infty$) and arbitrary thickness ($0\leqslant t/b\leqslant 0.8$). Fig. 4 of reference [13] gives data on the odd-mode fringing capacitance which can be used to determine Z_0 for the rectangular line with only small error. Getsinger quotes the accuracy of his data as being less than 1.24 percent provided that Cohn's criteria of $w/(b-t)\geqslant 0.35$ is adhered to. For narrower center conductors, edge interaction effects again become significant; for this case, the curves of Riblet [10] can be used to estimate the interaction correction term ΔC_f for values of $a/b\geqslant 2$.

C. Exact Solution for Rectangular Coaxial Lines with $t/b>0.2$

Although not the principal subject of this paper, it is worth noting that exact solutions have been derived for a particular class of rectangular line having a relatively thick center conductor that is approximately equidistant from the outer conductor (i.e., $a-w\cong b-t$) [15]-[17]. In Fig. 8 of Chen's paper [11], the fringing capacitance is

TABLE I
COMPARISON OF Z_0 VALUES FOR ZERO THICKNESS CASE ($t/b=0$)

Case		Tippet & Chang [9] Expressions (3) – (6)	Cruzan & Garver [18]		Iwakura & Arakawa [20]		Metcalf [19]	
a/b	w/b	Z_0	Z_0	Diff.	Z_0	Diff.	Z_0	Diff.
1	0.80	54.54	54.9 ± 1.0	+ 0.3	54.5 ± 0.5	0	54.5 ± 0.3	0
1	0.65	69.80	70.3 ± 0.7	+ 0.5	70.3 ± 0.7	+ 0.5	69.7 ± 0.3	- 0.1
1	0.5	87.03	87.0 ± 0.9	0	87.1 ± 1.7	+ 0.1	86.5 ± 0.4	- 0.5
2	1.80	34.54	34.6 ± 0.3	+ 0.1	34.0 ± 0.3	- 0.5	33.9 ± 0.2	- 0.6
2	1.50	45.07	45.2 ± 0.3	+ 0.1	45.2 ± 0.5	+ 0.1	44.7 ± 0.2	- 0.4
2	1.00	64.10	64.5 ± 0.8	+ 0.4	63.5 ± 2.5	- 0.6	63.0 ± 0.3	- 1.1
2	0.50	99.82	100.5 ± 1.5	+ 0.7	99.7 ± 7.0	- 0.1	97.0 ± 0.5	- 2.8

plotted as a function of inner conductor thickness for this particular case. Data for thin center conductors ($t/b<0.2$) are included in Chen's Fig. 8, thus giving the reader an impression that this solution is also valid for thin lines. This is not the case, since estimates of Z_0 obtained using Fig. 8 do not agree with the known exact values for $t/b=0$. However, Chen's data do appear to be accurate over the range $t/b>0.2$ or $t/(a-w)>0.5$ where the curve is approaching an asymptotic value of $C_f/\epsilon=0.559$ as $t/b\to 1$. Riblet [17] has obtained an exact solution for a one-parameter family of rectangular coaxial lines with $t/b=0.4$. For Riblet's 50-Ω line, $(a-w)/b=(b-t)/b=0.6$, thereby satisfying Chen's criteria. If the parameter values obtained by Riblet for w/b and a/b are substituted in Chen's equation (18) with $C_f/\epsilon=0.557$ (obtained from Chen's Fig. 8), the value obtained for Z_0 is in exact agreement with that of Riblet.

D. Numerical Techniques

A number of authors [18]–[20] have utilized numerical techniques to solve Laplace's equation for the generalized rectangular configuration including thick center conductors. Data for thin lines have generally been included. Cruzan and Garver [18] used an orthonormal block analysis technique and derived a series of nomographs giving values of corner capacitance C_f/ϵ against $a-w/(b-t)$ for various values of the parameters $2t/(a-w)$ and $2w/(b-t)$. The obvious complexity of the parameters utilized in this study somewhat limits the usefulness of the data derived. Furthermore, for lines with thin center conductors, much of the needed impedance data corresponds to relatively large values of $2w/(b-t)$ in the range 1 to 4. Such data are not given by Cruzan and Garver, thereby necessitating the use of extrapolation techniques in order to estimate the value of fringing capacitance. The error limits for their data are well defined by Cruzan and Garver. Metcalf [19] has used a relaxation method and published a very comprehensive set of design curves giving Z_0 as a function of t/b for different values of w/a and b/a. Data on the "slab" line of infinite width are also included. Metcalf claims an overall accuracy of better than 0.5 percent for his data. Iwakura and Arakawa [20] used a numerical integration technique and published some limited data in which Z_0 is plotted against w/a for the full range of t/b. Error limits are also defined in these plots, ranging from less than 1 percent to 5 percent or more.

III. COMPARISON OF DATA

It is instructive to compare the results obtained by numerical techniques with the exact data derived from the solution of Tippet and Chang [9]. Table I shows comparison data for the zero thickness case ($t/b=0$). The values chosen for the parameters a/b and w/b were those commonly considered by the three numerical technique authors. The numerical data require reading of graphical plots, so that an additional source of error is added which is estimated to be no more than ±0.25 Ω. In examining comparison data of Table I, it is apparent that the numerical values of Cruzan and Garver are consistently higher than the exact values, though the difference is small and within the error limits specified. The values of Metcalf are consistently lower, on the other hand, and the difference is frequently greater than the error limit specified by Metcalf.

Table II shows similar comparison data for the finite thickness case ($t/b=0.1$). Although none of these data are known to be exact, the values derived from Getsinger's data were felt to be the most accurate and were consequently used as a reference for comparison purposes. Where significant, the correction data of Riblet [10] were applied during the computation of the reference values. It is readily apparent from Table II that Chen's data deviate significantly from the other values, especially for $a/b=1$, so that it appears that (11) yields data of very questionable accuracy in the region $w/a>0.6$. It is also apparent that the values derived from Joines' expression (12) are, in almost all cases considered, consistently lower than the comparison values. The data of Cruzan and Garver [18] are again in very close agreement with the chosen reference values. A more valid comparison of characteristic impedance data cannot be undertaken until an exact solution is available for the problem involving center plates of finite but small thickness.

TABLE II
COMPARISON OF Z_0 VALUES FOR FINITE THICKNESS CASE ($t/b = 0.1$)

Case		Getsinger [13]	Chen [11] Expression (11)		Joines [12] Expression (12)		Cruzan & Garver [18]		Iwakura & Arakawa [20]		Metcalf [19]	
a/b	w/b	Z_0	Z_0	Diff.	Z_0	Diff.	Z_0	Diff.	Z_0	Diff.	Z_0	Diff.
1	0.80	40.97	44.7	+ 3.7	39.8	- 1.2	40.9 ± 0.3	- 0.1	40.7 ± 0.4	- 0.3	40.5 ± 0.2	- 0.5
1	0.65	55.17	57.4	+ 2.2	54.0	- 1.2	55.3 ± 0.1	+ 0.1	55.3 ± 0.6	+ 0.1	54.7 ± 0.3	- 0.5
2	1.8	27.62	29.3	+ 1.7	27.1	- 0.5	27.3 ± 0.1	- 0.3	27.7 ± 0.3	+ 0.1	27.7 ± 0.2	+ 0.1
2	1.5	38.10	38.7	+ 0.6	37.8	- 0.3	38.1 ± 0.05	0	38.0 ± 0.4	- 0.1	38.3 ± 0.2	+ 0.2
2	0.9	58.61	58.6	0	59.1	+ 0.5	58.8 ± 0.1	+ 0.2	58.6 ± 1.8	0	58.7 ± 0.3	+ 0.1
2	0.45	87.27	86.7	+ 0.6	87.0	- 0.3	87.2 ± 0.3	- 0.1	86.5 ± 5.2	- 0.8	85.9 ± 0.4	- 1.4

In conclusion, it is worth comparing the experimental results which Crawford [1] obtained for his TEM transmission cells with data predicted from (1) and (2) (these are assumed to have center plates of negligible thickness, since this is the only exact data available; actual values used by Crawford ranged from $t/b = 0.001$ to 0.009). Using the dimensional parameters specified by Crawford, the predicted values for Z_0 are 51.81 Ω for the square cells ($a/b = 1, w/b = 0.826$), 51.99 Ω for two of the rectangular cells ($a/b = 1.667$, $w/b = 1.202$), and 51.49 Ω for the third rectangular cell ($a/b = 1.667, w/b = 1.213$). These figures are in very close agreement with Crawford's design goal of a nominal 52-Ω characteristic impedance.

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